QUEEN'S UNIVERSITY

DEPARTMENT OF ECONOMICS

MIDTERM EXAMINATION

Economics 390

February 29, 2007 **Instructor:** John M. Hartwick

Time Allowed: 75 minutes.

General Instructions: Do I and Two others. (I is worth 40 marks and each other, 15).

I (a) What is PROFIT for an extractive firm?

(b) What is the relationship between PROFIT and the market value of the extractive firm?

(c) In the basic Hotelling Model of the competitive extractive industry, each firm earns the same profit. Explain.

(d) The value of the current remaining stock *S(t)* in the basic competitive Hotelling model is not the sum of the values of all remaining firms. Explain.

II What is NEUTRAL about a neutral tax? Explain in brief and illustrate.

III When Miller and Upton test Hotelling theory of extraction with data for firms from the New York Stock Exchange, they handle the different qualities of reserves for different firms poorly. Explain.

IV "Normal" extraction costs for many L.C. Gray firms in a competitive extractive industry will result in a market failure. Explain.

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PART A

Profit for an extractive firm in a competitive industry is equal to the difference between total revenue and total cost. ~~for all periods, discounted to the present period.~~

An extractive firm faces a finite resource stock. Thus, at some point in time, extraction will come to an end. The extractive firm tries to maximize the profit from this finite stock. However, since it is finite, extraction in each period affects every other period. Thus, they try to max PV of profits (for all periods) to max wealth:

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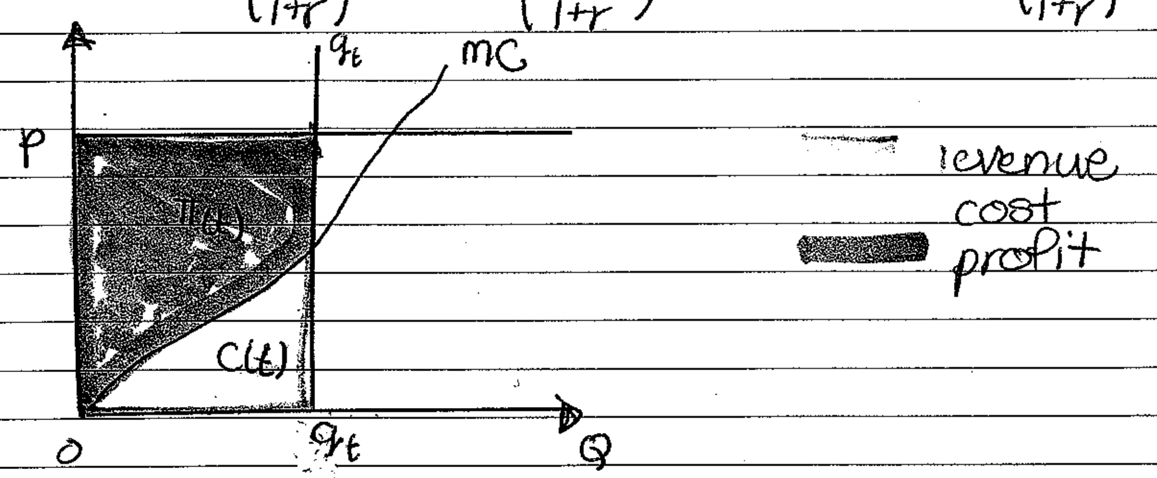
$$\pi(q_t) = P \cdot q_t - C(q_t)$$

profit in period t; price; quantity extracted in period t; costs of extraction (rising as q rises)

Then, given this, the problem for the firm is

~~PART B~~

$$\max \ \pi_0 + \left(\frac{1}{1+r}\right)\pi_1 + \left(\frac{1}{1+r}\right)^2 \pi_2 \ \ldots\ldots + \left(\frac{1}{1+r}\right)^T \pi_T$$



PART B

Market value of the firm is equal to the discounted value of its future profits flows.

$$V(S_t) = \pi(q_t) + \left(\frac{1}{1+r}\right)^{t+1} \pi(q_{t+1}) + \ldots\ldots + \left(\frac{1}{1+r}\right)^{T} \pi(q_T)$$

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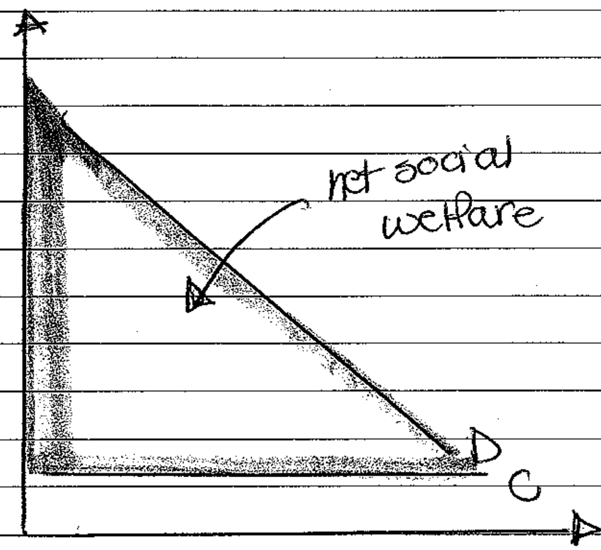
Basically, ~~it is the value of the~~ the market value of the firm is equal to the value of the remaining stock left in the ground. The value of that remaining stock left in the ground is what can be earned from it – that is, the PV of ~~its~~ the future profits it can earn.

$$q_t + q_{t+1} + \ldots + q_T = S_t$$

PART C

Each firm earns the same profit in the basic Hotelling model of the industry in order for efficient extraction to occur.

At the industry level, extraction must follow the r% rule. What moves @ r% is rent, because the industry tries to maximize net social welfare.



max W

max $B(Q_1) - cQ_1 + \left(\frac{1}{1+r}\right)$
$[B(Q_2) - cQ_2) \ldots$
$\left(\frac{1}{1+r}\right)^T [B(Q_T) - cQ_T$

r% rule is
$B(Q_t) - c(Q_t) = \left(\frac{1}{1+r}\right)[B(Q_{t+1}) - c(Q$

- This simplifies to $p_t - c = \left(\frac{1}{1+r}\right)(p_{t+1} - c)$

Price - MC = rent, so rent moves @ r% for efficient extraction.

$q_t = 1$ small

- For extraction to be efficient, each individual firm must be following L.C. Gray's model of extraction for the firm.

That is
$$MP_1 = \left(\frac{1}{1+r}\right) MP_2$$
for any 2 consecutive periods.

- What happens in the industry model is that, because p is not constant, $p_t - c$ will be different in different periods.

- Thus, the firm will want to pick the period where $p_t - c$ is the greatest.

- However, p_t is dependent on Q_t, since we have a demand schedule for the industry.

- Thus, p_t will adjust based on how much Q_t is put in that period.

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- In essence, firm profits will have to be the same in each period if rent is moving @ r%, because then the firm's will be indifferent between each period.

  - If you extract early, π_t will grow @ r%.
  - if you extract late, when less is being extracted, price will be higher, but it won't have the opportunity to grow @ r%.

- ~~Thus, this will make firms get the~~

- Thus, the demand schedule will make it such that firms earn the same profit, regardless of when they choose to extract. This will make the firm's indifference about when they extract

PART D

~~This is true. In the basic Hotelling model, not all firms will be extracting. Firms will remain, but will not necessarily be extracting.~~

~~$V(S_t) = \pi(Q_t) \cdot c(Q_t) \cdot t$~~

The firm & the industry face different cost functions, with affects the value of their profits or remaining stock. The value of the currently remaining stock in the competitive industry, as such, will differ from the value of the currently remaining firms, when their values are summed.

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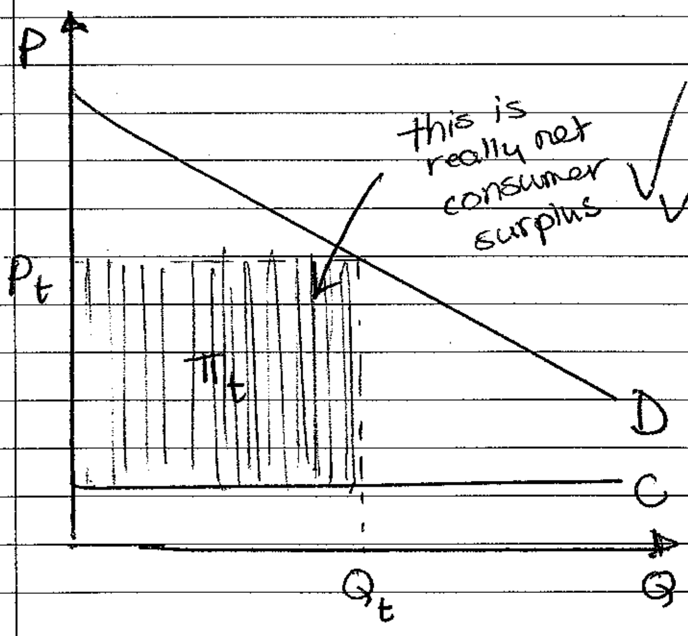
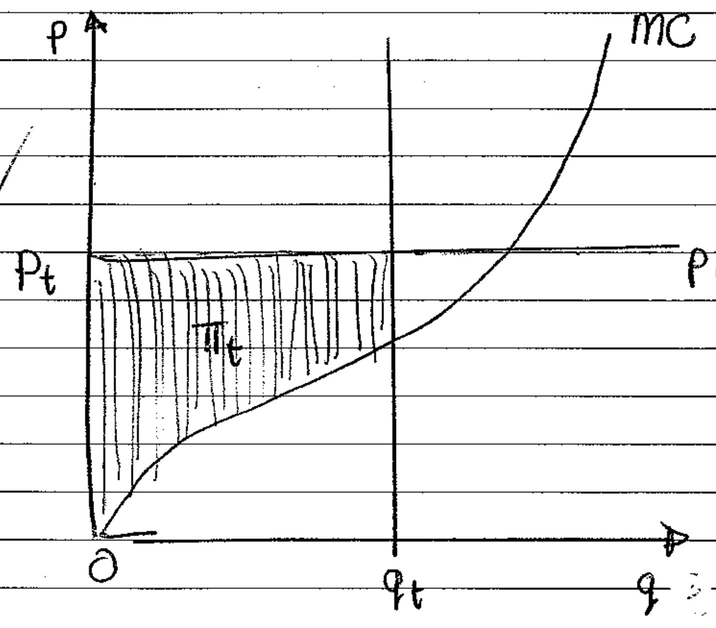


industry



firm

$$\underset{\text{industry}}{V(S_t)} = \pi_t + \left(\frac{1}{1+r}\right)^{t+1} \pi_{t+1} + \dots + \left(\frac{1}{1+r}\right)^{T} \pi_T$$

$$= (p_t \cdot Q_t - c\,Q_t) + \left(\frac{1}{1+r}\right)^{t+1} (p_{t+1} \cdot Q_{t+1} - c\,Q_{t+1})$$

$$\ldots \left(\frac{1}{1+r}\right)^{T}\left[p_T \cdot Q_T - cQ_T\right]$$

- $Q_T = \Sigma q_t$

$$= p\,\Sigma q_t - c\,\Sigma q_t + \left(\frac{1}{1+r}\right)^{t+1}\left[p\,\Sigma q_{t+1} - c\,\Sigma q_{t+1}\right] \ldots$$

- but price is a function of total industry extraction - so it changes - is not constant
- and cost is different as extraction changes
  → ~~increases as~~ q_t ↑
- sum of $V(S_t)$ for q_ts

$$= \sum \left([pq_t - C(q_t)] + \left(\frac{1}{1+r}\right)^{t+1}[pq_{t+1} - C(q_{t+1})] \ldots_T \right)$$

- these two things are simply not equal
  - cost functions differ

$$c \cdot Q_t \quad \text{vs.} \quad C(q_{it})$$

constant unit cost as quantity rises — increasing cost w. increasing quantity

Since we are sort of maximizing different things - social welfare vs. profit - it would make sense that they don't come out as being the sum of one another.

Also, since the cost functions are different (I know - I did a really poor job of showing it), profit for the firm will come out differently, so you cannot have $V(S_t) = \Sigma V(S_t^q)$.

QUESTION II

A ~~neutral~~ tax has no impact on the firm's profits or taxation path. For the extractive firm, Samuelson taxation leads to neutral taxation on the profits of an extractive firm.

Samuelson taxation has 2 components.

① Must allow for economic depreciation
- depletion in the value of the firm
- $D_t = V_t - V_{t+1}$ ✓

② Must allow for bond interest payment deducation
- must be able to deduct interest payments from your taxation.

r goes to $r(1-\tau)$

PV of profits$_t$

$$= \pi_t + \left(\frac{1}{1+r}\right)^{t+1} \pi_{t+1} + \ldots + \left(\frac{1}{1+r}\right)^T \pi_T$$

With a ~~...~~ tax rate of τ,

$$\text{Taxes} = \tau\left[\pi_t - D_t\right]$$

$$\text{Profit after tax} = (1-\tau)\,\pi(q_t) + \tau D_t$$ ✓

- so, PV of profits looks like

$$\text{PV of profits}_t = (1-\tau)\pi_t + \tau D_t + \left[\frac{1}{1+r(1-\tau)}\right]^{t+1}\left[(1-\tau)\pi_{t+1} + \tau D_{t+1}\right]$$

$$\ldots\ldots + \left[\frac{1}{1+r(1-\tau)}\right]^{T}\left[(1-\tau)\pi_T + \tau D_T\right]$$

- however, this has <u>no</u> impact on the firm's decision to extract
- $\pi_t = \pi_t^{\tau}$
  - PV of profits is unchanged
- what happens is

$$\left(\frac{1}{1+r}\right) > \left(\frac{1}{1+r(1-\tau)}\right)$$

so, the ~~value~~ discount rate is smaller
- means PV of profits in future goes up and profits end up being the same w. taxation
- with taxation, however, the firm will take longer to extract
  - $r > r(1-\tau)$
  - when $r \downarrow$, we know that extraction slows
- however, profits are unchanged and so the value of the company is unchanged
- this leads to neutrality
  - does not affect the firm's decision to produce
- furthermore, if we look @ $\frac{dV_t}{dt}$ → we find that all the τ's cancel out
  → no change

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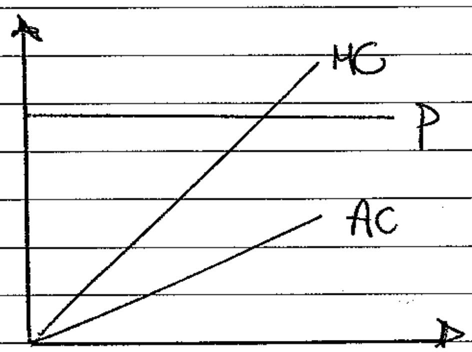
~~Neutral taxation, ideally, would not change the~~

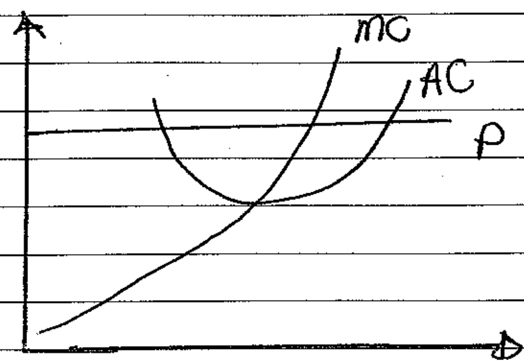
Thus, taxation of extraction profits is neutral if Samuelson taxation (depletion allowance & deduction of interest payments) is used.

PART IV

Normal extraction costs imply that MC cuts AC @ its minimum.



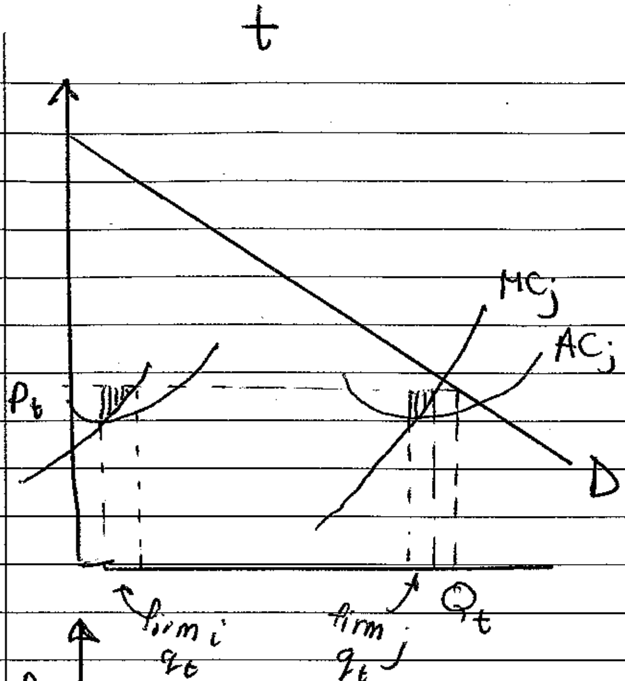
· what Gray used

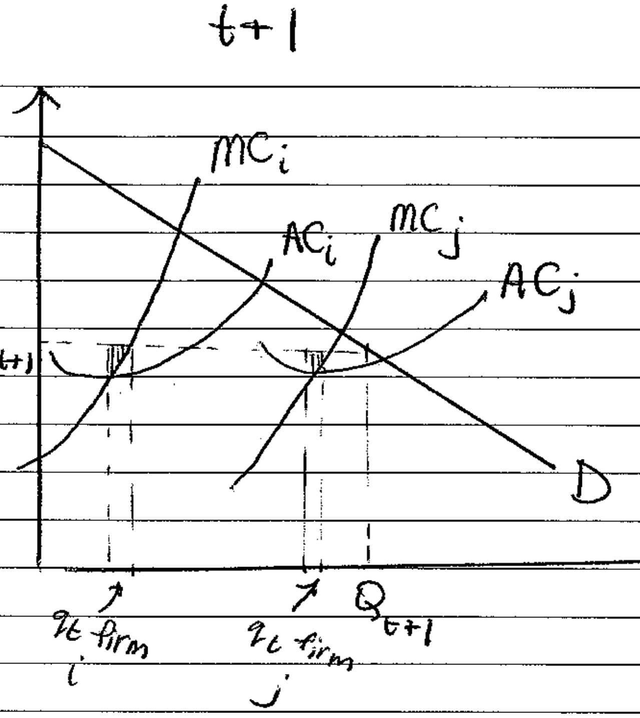


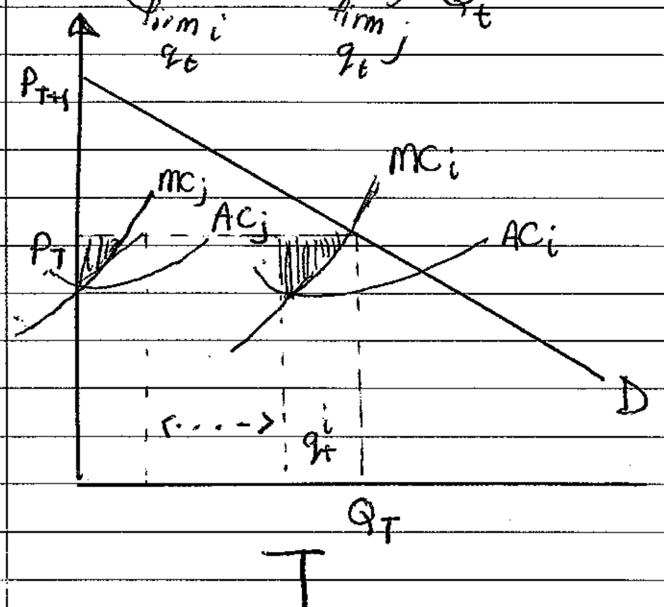
· what is "normal" and common for extractive firms.

Normal extraction costs lead to a declining AC. Declining AC means that it cannot be a perfectly competitive market, because some economies of scale will exist.

So, if we have these firms in a competitive extractive industry, they will extract until MC = AC (because that is where their MP = AP → $Q_T^{*} = 0$ condition exists).







- final period
- $MC = AC$
- q_t gets smaller & smaller in each period (even if my graph's don't show it too well)

- thus, in the final period, it will be too costly to extract any more oil (there may be some left)
- in the period after T, T+1, price will be equal to p_{T+1}, since Q_{T+1} will be equal to 0

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- however, the firms will know this
- will want to hoard oil to wait for T+1 to take advantage of p_{T+1}

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- as such, there is an incentive to stockpile oil
- no firm will follow this extraction path
- all will hoard to wait for the price jump

- the competitive solution will fall apart
- we would not get this if we had Gray's cost curve
- results from these "normal" extraction costs